DSP

19/10/2015 الاثنين

تمكشه $\boxed{3}$ م. شهاب

* Convolution Properties :-

① Commutation

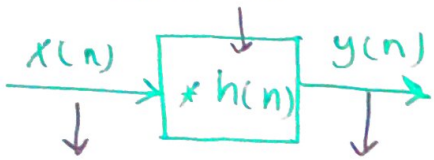
② Causality

$h(n)=0, \quad n<0$

③ Stability

$$\sum_{-\infty}^{\infty} |h(n)| < \infty$$

لازم يكون bounded



Stable : Bounded I/P — Bounded O/P

* for stability $h(n)$ is bounded ✗ ← غير كافي

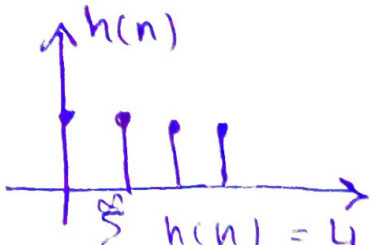
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

✓✓ # for stability

$$\sum_{n=-\infty}^{\infty} h(n) < \infty$$

ex:

$$\sum_{-\infty}^{\infty} h(n) = \infty$$



$$\sum_{-\infty}^{\infty} h(n) = 4$$

ex: $h(n) = u(n) - u(n-a)$

$$\sum_{-\infty}^{\infty} h(n) = a$$

$\Rightarrow$ stable

ex: $h(n) = u(n)$

$$\sum_{-\infty}^{\infty} h(n) = \infty$$

$\Rightarrow$ unstable

* Z-transform

على الهامش

Continous $\longrightarrow$ Biscrete

differential Equations $\left(\frac{d}{dt}\right)$ — difference Equation (Δ)

for Higher orders $\downarrow$ Laplace Transform

$\downarrow$ Z-transform

$$L[f(t)] = F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt$$

$$F(s) = \sum_{0}^{\infty} f(nT)\, e^{-snT}$$

$$Z = e^{sT}$$

$$F(Z) = \sum_{0}^{\infty} f(n)\, Z^{-n}$$

Sheet :

① $x(n) = n e^{3n} u(n)$

find $X(z)$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} n e^{3n} z^{-n}$$

$$= e^3 z^{-1} + 2e^6 z^{-2} + 3e^9 e^{-3} + 4e^{12} z^{-4} + \cdots$$

$$= e^3 z^{-1} \left(1 + 2e^3 z^{-1} + 3e^6 z^{-2} + 4e^9 z^{-3} + \cdots\right)$$

$$= e^3 z^{-1} \left(1 + 2e^3 z^{-1} + 3(e^3 z^{-1})^2 + 4(e^3 z^{-1})^3 + \cdots\right)$$

$$X(z) = \frac{e^3 z^{-1}}{(1 - e^3 z^{-1})^2}$$

Another Solution

$$X(z) = e^3 z^{-1} + 2e^6 z^{-2} + 3e^9 z^{-3} + \cdots$$

$$e^3 z^{-1} X(z) = e^6 z^{-1} + 2e^9 z^{-2} + 3e^{12} z^{-3} + \cdots$$

$$X(z) - e^3 z^{-1} X(z) = e^3 z^{-1} + e^6 z^{-2} + e^9 z^{-3} + \cdots$$

$$= e^3 z^{-1} \left(1 + e^3 z^{-1} + e^6 z^{-2} + \cdots\right)$$

$$(1 - e^3 z^{-1}) X(z) = e^3 z^{-1} \frac{1}{1 - e^3 z^{-1}}$$

$$X(z) = \frac{e^3 z^{-1}}{(1 - e^3 z^{-1})^2}$$

② $x(n) = (-1)^n u(n)$

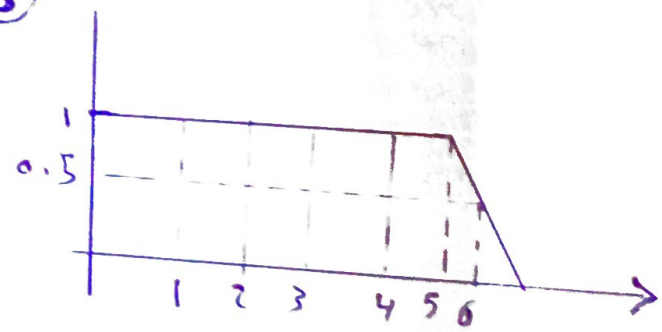
$$X(z) = \sum_{0}^{\infty} (-1)^n z^{-n}$$

$$= 1 + (-1) z^{-1} + (-1)^2 z^{-2} + (-1)^3 z^{-3} + \cdots$$

$$= 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - \cdots$$

$$= \frac{1}{1 + z^{-1}}$$

③

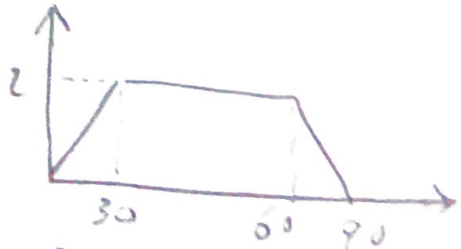


$x(n) = \{1, 1, 1, 1, 1, 0.5, 0\}$

$$X(z) = \sum_{0}^{\infty} x(n) z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + 0.5 z^{-5}$$

Report



find $X(z)$ $T = 1$

Properties of Z-transform

1. $Z[a\,x(n)] = a\,X(z)$
2. $Z[x_1(n) + x_2(n)] = X_1(z) + X_2(z)$
3. $Z[e^{\pm an}\,x(n)] = X(z)\big|_{z=ze^{\mp a}}$ (عكس الإشارة)
4. $Z[n\,x(n)] = -z\frac{d}{dz}X(z)$
5. $Z[a^n\,x(n)] = X(z)\big|_{z=z/a}$
6. $Z[x_1(n) * x_2(n)] = X_1(z).X_2(z)$ (convolution)
7. $Z[x(n \pm m)] = z^{\pm m}X(z)$ (الإشارة لا تتغير)

$x(n) = (0.5)^n + (0.25)^n + 3\,\delta(n-2)$

Find $X(z)$

$x(n) = (0.5)^n u(n) + (0.25)^n u(n) + 3\,\delta(n-2)$

$$X(z) = \frac{z}{z-1}\Big|_{z=\frac{z}{0.5}} + \frac{z}{z-1}\Big|_{z=\frac{z}{0.25}} + 3z^{-2}$$

$$= \frac{z}{z-0.5} + \frac{z}{z-0.25} + 3z^{-2}$$

$x(n) = (-1)^n \sin(\omega n) + e^{-2n}\cos(\omega n)$

$$X(z) = Z[\sin(\omega n)]\Big|_{z=-z} + Z[\cos(\omega n)]\Big|_{z=ze^{2}}$$

(حوض)

$x(n) = (-1)^n u(n)$

$$X(z) = U(z)\Big|_{z=-z}$$

$$= \frac{z}{z-1}\Big|_{z=-z}$$

$$= \frac{-z}{-z-1}$$

$$= \frac{z}{z+1} = \frac{1}{1+z^{-1}}$$

$$X(s) = \frac{10}{(s+1)(s+2)}$$

$$X(s) = \frac{10}{s+1} - \frac{10}{s+2}$$

$$L^{-1}[X(s)] = x(t) = 10e^{-t} - 10e^{-2t}$$

$\Rightarrow$ sampling $t = nT$, $T = 1$

$$x(n) = 10e^{-n} - 10e^{-2n}$$

$$X(z) = 10\left[\frac{z}{z-1}\Big|_{ze^{n}} - \frac{z}{z-1}\Big|_{ze^{2n}}\right]$$

$$z = e^{sT}$$

$$\ln z = sT$$

$$X(z) = \frac{10}{(\ln z + 1)(\ln z + 2)}$$

$$y(n) = x_1(n) * x_2(n)$$

$$x_1(n) = (0.5)^n u(n-1)$$

$$x_2(n) = \delta(n) e^{2n}$$

Find $X(z)$

$$X(z) = X_1(z) \cdot X_2(z)$$

$$X_1(z) = z\left[u(n-1)\right]\Big|_{z=\frac{z}{0.5}}$$

$$= z^{-1} z\left[u(n)\right]\Big|_{z=\frac{z}{0.5}}$$

$$= \frac{z}{z-1}\Big|_{z=\frac{z}{0.5}=2z}$$